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Series RCL Circuit

Consider the *RCL* circuits as shown in figure 3.9. Each has a low-frequency and high-frequency approximation. Considering the band-reject filter (figure 3.6d) we obtain for the transfer function

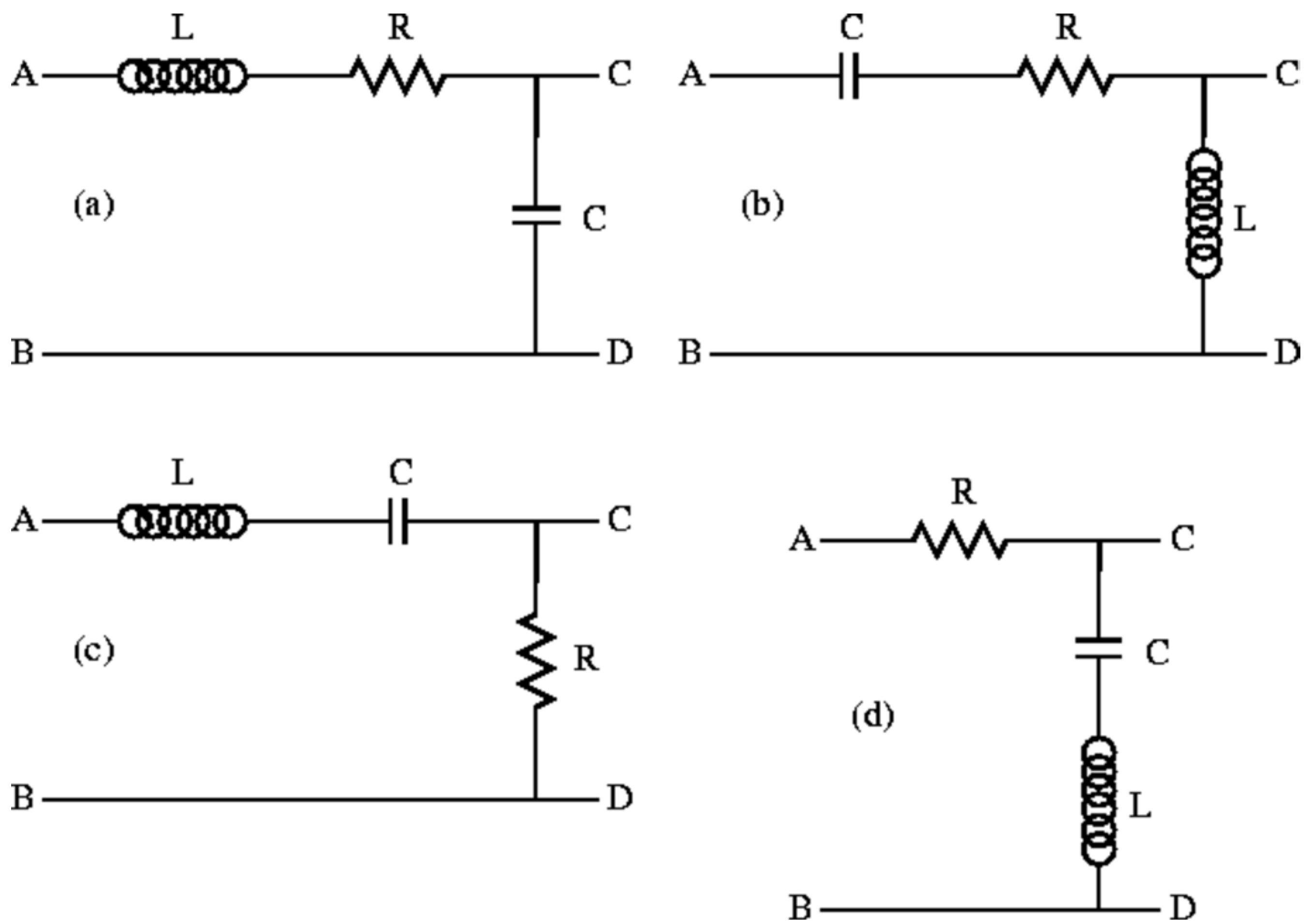

Figure 3.9: LCR filters: a) low-pass, b) high-pass, c) band-pass and d) band-reject.

$$\mathbf{H} = \frac{1/(j\omega C) + j\omega L}{R + 1/(j\omega C) + j\omega L} \quad (3.51)$$

$$= \frac{1 - \omega^2 LC}{(1 - \omega^2 LC) + j\omega RC}. \quad (3.52)$$

The approximations are:

$$\omega \to 0; \quad \mathbf{H} \to \mathbf{H}_L = 1 \quad \text{and} \quad (3.53)$$

$$\omega \to \infty; \quad \mathbf{H} \to \mathbf{H}_H = 1. \quad (3.54)$$

We notice a zero in the transfer function at $\omega_0 = 1/\sqrt{LC}$. In the low-medium frequency range

$$\omega < \omega_0; \quad \mathbf{H} \to \mathbf{H}_{LM} = \frac{1}{j\omega RC} \propto \omega^{-1}, \quad (3.55)$$

for high-medium frequencies

$$\omega > \omega_0; \quad \mathbf{H} \to \mathbf{H}_{HM} = \frac{-\omega^2 LC}{j\omega RC} = \frac{j\omega L}{R} \propto \omega^{+1}. \quad (3.56)$$

Solving for the corner frequencies we have

$$1 = 1/|j\omega RC| \quad \Rightarrow \quad \omega_1 = 1/(RC), \tag{3.57}$$

$$1 = |j\omega L/R| \quad \Rightarrow \quad \omega_2 = R/L \quad \text{and} \tag{3.58}$$

$$\omega_0 = 1/\sqrt{LC} = \sqrt{\omega_1\omega_2}. \tag{3.59}$$

Example: *Sketch* $|\mathbf{H}(j\omega)|$ *for the LCR circuit shown in figure 3.10 for the two conditions* $R = 0.5\sqrt{L/C}$ *and* $R = 2\sqrt{L/C}$. *In each case, determine the values of* $|\mathbf{H}|$ *at* $\omega = 0$, ∞, *and* ω_c, *and label these points on the sketches.*

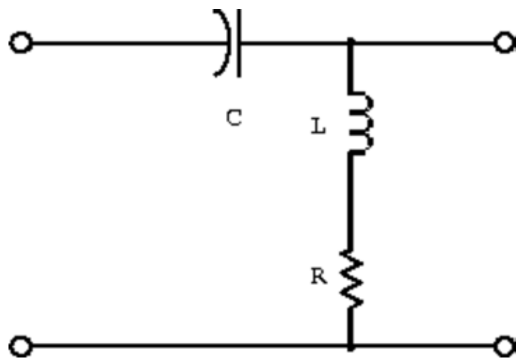

Figure 3.10: *LCR circuit with two components across the output.*

The transfer function is

$$\mathbf{H}(j\omega) = \frac{R + j\omega L}{R + j\omega L + \frac{1}{j\omega C}}. \tag{3.60}$$

For ω *small*

$$\mathbf{H}(j\omega) \approx \frac{R}{1/(j\omega C)} = j\omega RC \tag{3.61}$$

$$|\mathbf{H}(j\omega)| = \omega RC. \tag{3.62}$$

For large ω *:* $\mathbf{H}(j\omega) = 1$.

For the corner frequency: $1 = \omega_C RC \rightarrow \omega_C = 1/(RC)$.

For $R = 0.5\sqrt{L/C} = \frac{1}{2}\sqrt{\frac{L}{C}}$,

$H_{low} = \frac{\omega}{2}\sqrt{LC}$; $\omega_C = \frac{2}{\sqrt{LC}}$.

$$\mathbf{H}(j\omega_C) = \frac{R + 2j\sqrt{L/C}}{R + 2j\sqrt{L/C} - j/2\sqrt{L/C}} = \frac{R + 4jR}{R + 4jR - jR} \tag{3.63}$$

$$= \frac{1 + 4j}{1 + 3j} = \frac{(1 + 4j)(1 - 3j)}{1 + 9} = \frac{13 + j}{10} \tag{3.64}$$

$$|\mathbf{H}(j\omega_C)| = \sqrt{\frac{13^2 + 1^2}{10^2}} = \sqrt{\frac{170}{10}} = 1.30. \tag{3.65}$$

For $R = 2\sqrt{L/C}$,

$H_{low} = 2\omega\sqrt{LC}$; $\omega_C = \frac{1}{2\sqrt{LC}}$

$$\mathbf{H}(j\omega_C) = \frac{R + j/2\sqrt{L/C}}{R + j/2\sqrt{L/C} - 2j\sqrt{L/C}} = \frac{2 + j/2}{2 + j/2 - 2j} \quad (3.66)$$

$$= \frac{4+j}{4-3j} = \frac{(4+j)(4+3j)}{16+9} = \frac{13+16j}{25} \quad (3.67)$$

$$|\mathbf{H}(j\omega_C)| = \sqrt{\frac{13^2+16^2}{25^2}} = \frac{\sqrt{425}}{25} = 0.825. \quad (3.68)$$

Figure 3.11 is a sketch of the transfer functions.

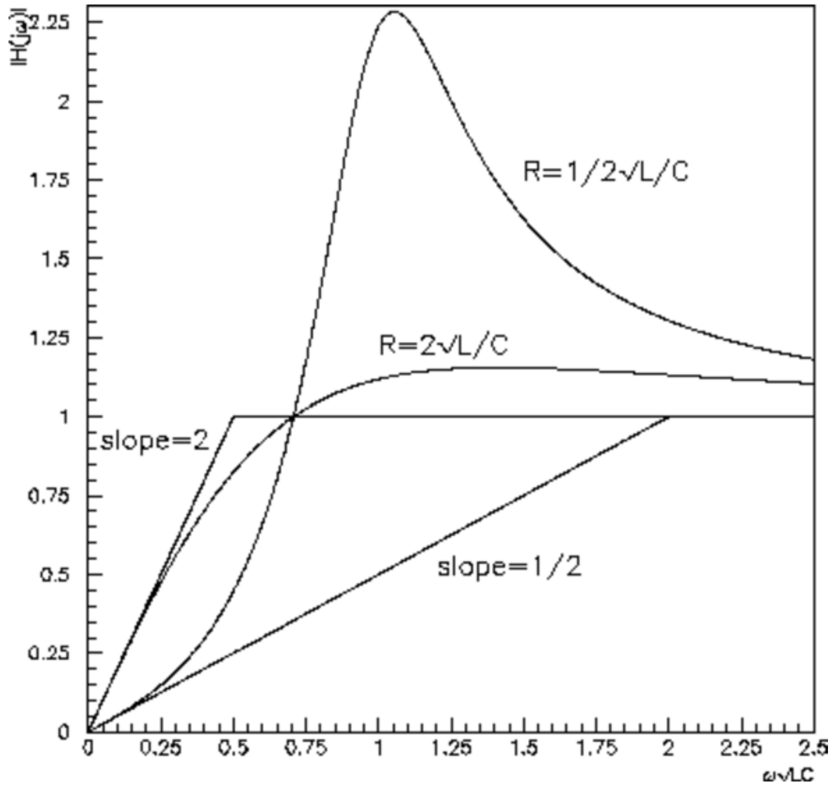

Figure 3.11: *Sketch of the transfer functions for the above circuit.*

Example:

1. *Write an expression for the transfer function of the circuit shown in figure 3.12.*

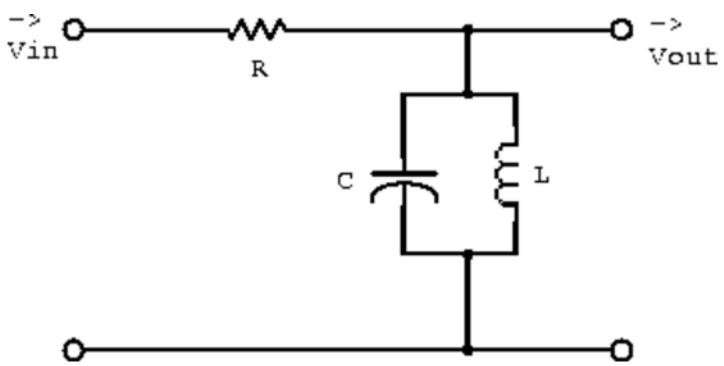

Figure 3.12: *Circuit with components in parallel at the output.*

$$Z_{CL} = \frac{(1/(j\omega C))(j\omega L)}{1/(j\omega C) + j\omega L} = \frac{j\omega L}{1 - \omega^2 CL} \quad (3.69)$$

$$\mathbf{H}(j\omega) = \frac{\frac{j\omega L}{1-\omega^2 CL}}{R + \frac{j\omega L}{1-\omega^2 CL}} = \frac{1}{1 - jR/(\omega L)(1-\omega^2 CL)} \quad (3.70)$$

$$= \frac{1}{1 + j\left(\omega RC - \frac{1}{\omega}\frac{R}{L}\right)}. \quad (3.71)$$

2. *What phase shift is introduced by this filter at very small and very large frequencies?*

   For large ω $\mathbf{H}(j\omega) \approx \frac{1}{j\omega RC} = \frac{-j}{RC}\omega^{-1}$

$$\mathbf{H}_{high} = \frac{1}{RC}\omega^{-1} e^{-j\pi/2} \rightarrow \phi_H = -\frac{\pi}{2}. \quad (3.72)$$

   For small ω $\mathbf{H}(j\omega) \approx \frac{1}{(-j/\omega)(R/L)} = j\omega\frac{L}{R}$

$$\mathbf{H}_{low} = \frac{L}{R}\omega e^{j\pi/2} \rightarrow \phi_L = +\frac{\pi}{2}. \quad (3.73)$$

3. *On a log-log scale, sketch* $|\mathbf{H}(j\omega)|$ *and the phase shift as a function of* ω .

   For the corner frequency $\omega_C^2 RC = \frac{R}{L} \rightarrow \omega_C = \frac{1}{\sqrt{LC}}$.

   $\mathbf{H}(j\omega_C) = 1; \phi_C = 0$.

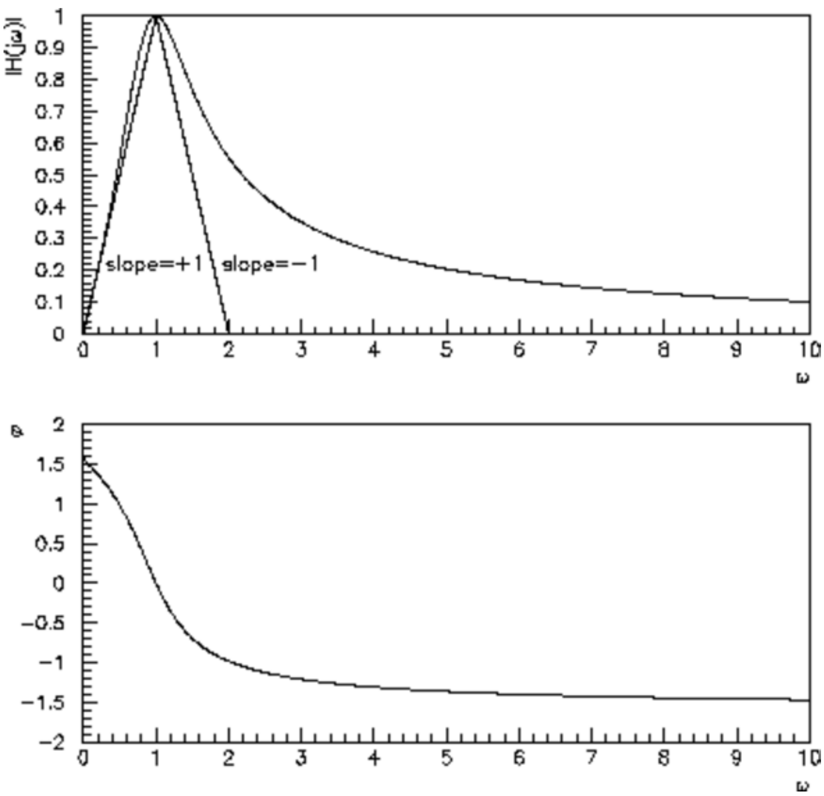

Figure 3.13: *Transfer function and phase shift for the above circuit.*

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